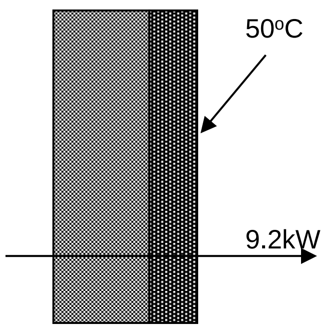
University of Nottingham
Department of Mechanical, Materials and Manufacturing Engineering
Thermofluids 3
Solutions to Exercise sheet 1: Conduction (Revision)

1. A composite wall has 2 layers in perfect thermal contact. One is 20cm thick and of uniform thermal conductivity k = 2.8 W/mK. The other is 10cm thick and has k = 0.7 W/mK. Calculate the thermal resistance of the wall assuming one-dimensional conduction and, given that the rate of heat conduction through the wall is 9.2kW/m^2 and that the temperature of the outer surface of the 10cm layer is 50^{o}C, calculate the temperature at the outer surface of the 20cm layer and at the interface.

Solution



For I-D conduction in a homogeneous medium $q = kA\dfrac{\Delta T}{\Delta x}$

We are not given the surface area so instead we will consider a 1m^2 section of the wall. Thermal resistance

$$R = \Sigma\left(\frac{\Delta x}{kA}\right) = \frac{0.2}{2.8} + \frac{0.1}{0.7} = 0.0714 + 0.1429 = 0.2143\ K/W$$

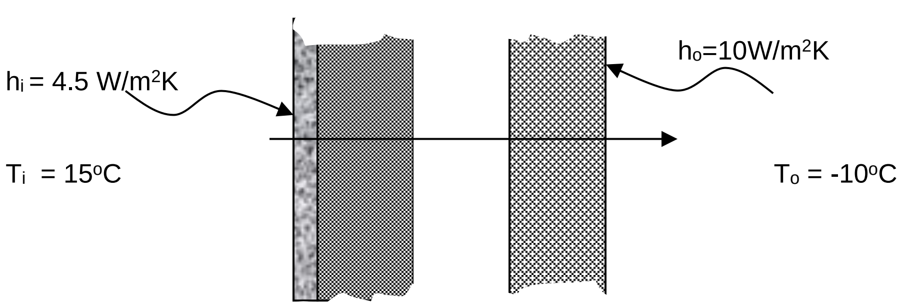
$\therefore$ Overall wall temperature drop = 920030.2143 = 1971K
Outer surface temperature of 20cm layer =1971+ 50 = <u>2021^{o}C</u>

Interface temperature = 50 + 920030.1429 = <u>1365^{o}C</u>

2. The cavity wall of a room has 2 layers of brick, each 11cm thick, separated by an 11cm air gap. The inner (room) surface of the inner layer is plastered to a thickness of 2cm. Heat transfer is by convection only on all exposed surfaces and by conduction only through the solid layers.
   (a) Calculate the heat lost per unit area of the wall and the temperature of each surface given that the room air temperature is 15^{o}C and the ambient air temperature is –10^{o}C.
   (b) Calculate also the reduction in heat loss due to completely filling the air gap with urethane foam.

<u>Data</u>
Thermal conductivities:
Inner brick layer: 0.69 W/mK; Outer brick layer: 1.32 W/mK; Plaster: 0.48W/mK; Foam: 0.018 W/mK
Heat transfer coefficients:
Room air–to-plaster: 4.5 W/m^{2}K; Brick-to-air gap air: 4.5 W/m^{2}K; wall-to-ambient air: 10 W/m^{2}K

Solution



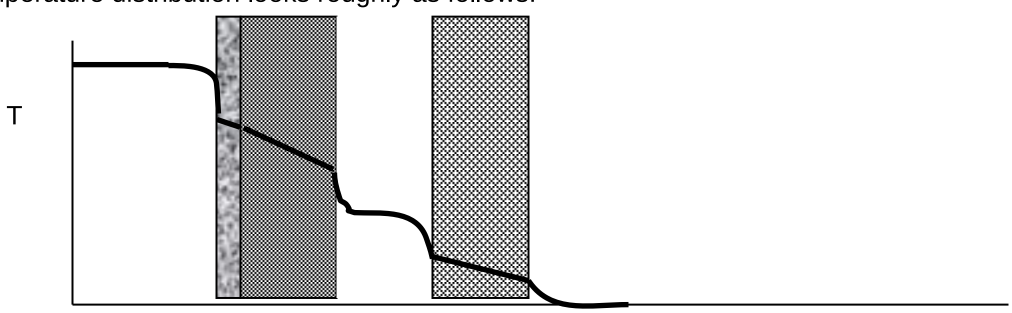
Model as resistances in series:

Overall resistance R = 1.051 K/W
∴ Heat flux $q = \Delta T/R = (15- (-10))/R = 25/1.051 =$ <u>23.8 W/m²</u>

Using $\Delta T = q R$, temperature drops across resistances are to 1 dec place (in order of above diagram):

5.3	1	3.8	5.3	5.3	2	2.4

so the temperature distribution looks roughly as follows:-



For a urethane foam filled cavity, the two (convection) resistances of 1/4.5, are replaced by a single (conduction) resistance of 0.11/0.018.
The overall resistance is then 6.718 K/W so the heat flux falls to 3.72 W/m²

3. A high-pressure steam pipe of diameter 28cm is lagged with 2 layers of insulation, the first being of thickness 5cm and thermal conductivity 0.086 W/mK. For the second (outer) layer, the corresponding values are 7cm and 0.06 W/mK. The pipe wall temperature is 500°C and the ambient air temperature is 25°C.
The heat loss from the outer surface of the lagging is by natural convection for which the heat transfer coefficient is given by:

$$h = 2.23\left(\frac{T_{surface} - T_{ambient}}{D}\right)^{1/4}$$

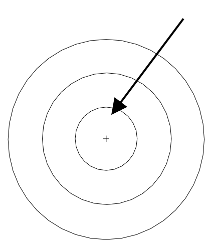
where D cm is the outside diameter of the lagging.

Calculate $T_{surface}$ and the heat loss/unit length of pipe (needs iterative/graphical solution).

Calculate also the overall heat transfer coefficient based on the exposed lagging surface area.

Solution

$r_1 = 0.14$



$T_{air} = 25°C$

Thermal resistance pipe-to-air per unit length of pipe

$$R = \left(\frac{\ln\left[r_2/r_1\right]}{2\pi k_{12}}\right) + \left(\frac{\ln\left[r_3/r_2\right]}{2\pi k_{23}}\right) + \frac{1}{2\pi r_3 h}$$

$r_1 = 0.14$m; $r_2 = 0.19$m; $r_3 = 0.26$m and $k_{12} = 0.086$ W/mK; $k_{23} = 0.06$ W/mK

h is obtained from the given correlation as a function of the surface temperature of the lagging, T_3, so R is a function of T_3.
Since we will write q = (T_3-T_{air})/R to calculate the heat loss, we will need to guess an initial value of T_3 so that h and thus R can be calculated. We can then use q to get a better estimate of T_3 and repeat the calculation until convergence is achieved:

e.g. Guess T_3 = 50°C. Then $h = 2.23\left(\frac{50-25}{52}\right)^{1/4} = 1.86$ W/m^{2}K

then $$R = \left(\frac{\ln 19/14}{2\pi \times 0.086}\right) + \left(\frac{\ln 26/19}{2\pi \times 0.06}\right) + \frac{1}{2\pi \times 0.26 \times 1.86} = 1.397 + 0.329 = 1.726 \text{ K/W}$$

The heat loss from the pipe is then (500-25)/1.726 = 275.2W/m

This gives the temperature drop from the lagging surface to the air as 275.2×0.329 =90.5 °C
So, the new T_3 = 115.5°C, giving h = 2.56 W/m^{2}K, R = 1.64 K/W, q = 290 W/m and T_3 =94.3°C.
Iterating again: h = 2.4 W/m^{2}K, R = 1.397+ 0.255 = 1.65 K/W, q = 287.5 W/m and T_3 = 98.3°C
This is close to the final answer.

A graphical method
Instead of computing the heat flow from the total thermal resistance, we can equally well calculate it as a function of T_3 using the thermal resistance of the lagging alone i.e. we write
$q = \frac{500 - T_3}{1.397}$ and hence we can plot q versus T_3 . (This is clearly a straight line).

We can also evaluate and plot a few corresponding values of q and T_3 on the same graph calculating q from the lagging-to-air temperature drop and thermal resistance as

$$q = 2\pi r_3 h(T_3 - T_{air}) = 2\pi \times 0.26 \times 2.23\left(\frac{T_3 - 25}{52}\right)^{1/4} \times (T_3 - 25)\ .$$

The heat conducted through the lagging must be equal to the heat convected from the lagging to the air
i.e. The solution is given by the point at which the two graphs cross.

Yet another alternative would be to combine the two equations above into a single non-linear equation for T_3 and hope that your PC has an adequate solver.

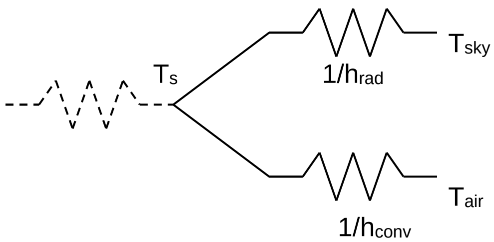
4. Recalculate the heat loss per unit area of wall for question 2 allowing for the additional heat loss due to radiation from the outer surface of the wall to the night sky for which the temperature is taken to be 20 degrees below the ambient air temperature. Take the case with an unfilled air gap. Assume that the radiation heat loss per unit area from the wall outer surface is given by:

$$q_{radiation} = 0.93\sigma(T_{surface}^4 - T_{sky}^4)$$

where the Stefan-Boltzmann constant σ = 5.67 3 10^{-8} W/m^{2}K^4 *and temperatures are* ***absolute***

Solution *– see next page*

As for question 3, an iterative or graphical solution is clearly required – the heat loss is a function of a temperature that depends on the heat loss.
The resistance chain is modified by the inclusion of a resistor to model the radiation loss. Since radiation occurs simultaneously with, and independent of, convection, the new resistor is in parallel with that representing convection resistance:



We also need to recognize that the heat is exchanged with regions differing in temperature

$$\dot{q}''_{rad} = 0.93\sigma(T_s^4 - T_{sky}^4) = 0.93\sigma(T_s + T_{sky})(T_s^2 + T_{sky}^2)(T_s - T_{sky})$$

$$\text{so} \quad h_{rad} = 0.93\sigma(T_s + T_{sky})(T_s^2 + T_{sky}^2)$$

Procedure is to guess a value of T_s, use this to calculate the sum of radiation and convection heat flows from the wall as:

$$\dot{q}''_{ext} = h_{rad}(T_s - T_{sky}) + h_{conv}(T_s - T_{air})$$

The relationship for the heat flow from the room to the outer surface of the wall

$$\dot{q}''_{int} = \frac{T_{room} - T_s}{\sum_{room}^{s} R} = \frac{15 - T_s}{0.951}$$

can then be used to calculate a new value for T_s (by recognizing that $\dot{q}''_{int} = \dot{q}''_{ext}$) and so on…….

T_{air} = -10°C so T_{sky} = -30°C = 243K
Without radiation, T_s was about 2.5° above ambient air temperature - guess that it will now lie close to air temperature i.e. take T_s = -10°C = 263K

Then $h_{rad} = 0.93 \times 5.67 \times 10^{-8} \times (263 + 243) \times (263^2 + 243^2) = 3.42\ \text{W/m}^2\text{K}$

so $\dot{q}''_{ext} = 3.42 \times 20 + 10 \times 0 = 68.4\ \text{W/m}^2$

and revised value of T_s = 15 - 0.951×68.4 = -50 °C = 223K

– clearly ridiculous as it cannot be < T_{sky}. Using this temperature would not be sensible as the external heat flow would then be from the sky and air to the wall!

A sensible move in cases like these is to resort to a graphical method to see where the solution is likely to lie.
Instead of using the second heat flow equation to evaluate T_s, we use it to evaluate q''_{int}
Thus q_{int}'' = (15- 10)/0.951= 5.26 W/m²
Reducing T_s would increase q''_{int} while reducing the loss calculated from the exterior network, q''_{ext}, thus bringing them closer into line.
So try now T_s = -15°C = 258K. This gives q''_{ext} = -0.485W and q''_{int} =31.6W

T_s	q''_{ext}	q''_{int}
-10	68.4	5.26
-15	-0.5	31.6

Even without plotting these, it is obvious that the solution must lie around -12.5°C. This gives q''_{ext} = 33.6W and q''_{int} = 28.9W
A little more work gives the final answer as T_s = -12.82°C and <u>q" = 29.25W</u>
{ **A point to ponder***: Why didn't iteration work?}*

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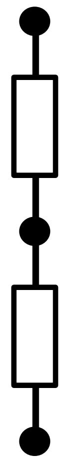
Assumptions and points

1. Assume that it is steady state
2. $\dot{q}''$ through skin layer is the same as $\dot{q}''$ through the air by convection because of 1.
3. R_{TH} for conduction by Fourier Law is $\Delta x/kA$
4. R_{TH} for convection is from Newton's Law $1/hA$
5. Total Thermal Resistance is $\Sigma\, R_{TH} = \frac{\Delta x}{kA} + \frac{1}{hA}$
6. Assume A=1m^2 . So calculating heat flow per unit area $\dot{q}''$
7. Once we have total resistance, we can get heat flow. This can be used with either resistances to get the temperature of the skin T_s



a)
5->

$$\Sigma R_{TH} = \frac{\Delta x}{kA} + \frac{1}{hA} = \frac{0.01}{0.42 * 1} + \frac{1}{30 * 1} = 0.0238 + 0.0333 = 0.0571\, K/W$$

7->

$$\dot{q}'' = \frac{36.5 - 20}{0.0571} = 289\, W/m^2$$

Using subskin layer to get T_s

$$\dot{q}'' = \frac{36.5 - T_s}{0.0238} \rightarrow T_s = 36.5 - 0.0238\, \dot{q}'' = 29.6\, C$$

So if the air temperature is 20C, the subskin layer is at 29.6C

b) WE can repeat the calculation with:

5->

$$\Sigma R_{TH} = \frac{\Delta x}{kA} + \frac{1}{hA} = \frac{0.01}{0.42 * 1} + \frac{1}{500 * 1} = 0.0238 + 0.002 = 0.0258\, K/W$$

7->

$$\dot{q}'' = \frac{36.5 - 10}{0.0258} = 1027 W/m^2$$

Use subskin layer again

$$\dot{q}'' = \frac{36.5 - T_s}{0.0238} \rightarrow T_s = 36.5 - 0.0238\, \dot{q}'' = 12\, C$$

c) $\frac{\dot{q}''_W}{\dot{q}''_A} = 3.5$

d) Assuming lumped capacitance, the rate of change of energy in thermal mass of the person is $mc\frac{\Delta T}{\Delta t}$

Energy balance states

$$mc\frac{\Delta T}{\delta t} = \Sigma\dot{q} + \dot{q}_{GEN}$$

1. $\dot{q}''$ from surface is $1027\ W/m^2$.
2. $q'' = \dot{1}00\ W$
3. $c = 1930\ J/kgK$

Rearrange energy balance to get :

$$\delta t = \frac{mc\ \Delta T}{\dot{q} + \dot{q}_{GEN}}$$

For stage 1: $\Delta T = 1.5\ C$

$$\delta t = \frac{(70)(1930)(1.5)}{1027 - 100} = 218\ s\ or\ 3.6\ min$$

For stage 2: $\Delta T = 3.5\ C$

$$\delta t = \frac{(70)(1930)(3.5)}{1027 - 100} = 510\ s\ or\ 8.5\ min$$

For stage 3: $\Delta T = 6.5\ C$

$$\delta t = \frac{(70)(1930)(6.5)}{1027 - 100} = 947\ s\ or\ 15.8\ min$$

This really requires a proper transient solution since the assumptions are very unrealistic.

C.f. rough time in air is longer (stage 1: ½ hr Stage 2: 0.7 hr Stage 3: 1.3 hr)